

The Pinax-Model for Accurate and Efficient Refraction Correction of Underwater Cameras in Flat-Pane Housings

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Abstract

A calibration and refraction correction process for underwater cameras with flat-pane interfaces is presented that is very easy and convenient to use in real world applications while yielding very accurate results. The correction is derived from an analysis of the axial camera model for underwater cameras, which is among others computationally hard to tackle. It is shown how realistic constraints on the distance of the camera to the window can be exploited, which leads to an approach dubbed Pinax Model as it combines aspects of a virtual pinhole model with the projection function from the axial camera model. It allows the pre-computation of a lookup-table for very fast refraction correction of the flat-pane with high accuracy. The model takes the refraction indexes of water into account, especially with respect to salinity, and it is therefore sufficient to calibrate the underwater camera only once in air. It is shown by real world experiments with several underwater cameras in different salt and sweet water conditions that the proposed process outperforms standard methods. Among others, it is shown how the presented method leads to accurate results with a single in-air calibration and even just estimated salinity values.

Keywords: underwater vision, calibration, rectification, refraction correction,

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1. Introduction

Cameras are very important sensors for underwater applications in general. This includes ship hull, pipeline and other inspection missions (Hollinger et al., 2012; Kim & Eustice, 2013; Foresti, 2001; Asakawa et al., 2000; Negahdaripour & Firoozfam, 2006; McLeod et al., 2013; Galceran et al., 2014), habitat mapping (Davie et al., 2008; Bodenmann et al., 2013), vehicle station-keeping (Negahdaripour & Fox, 1991; Marks et al., 1994; Lots et al., 2000), archeology (Bingham et al., 2010; Chapman et al., 2010; Hue et al., 2011) or search and recovery missions (Purcell et al., 2011) to just name a few examples - a short overview with respect to underwater vision on unmanned underwater vehicles (UUV), e.g., is given in (Horgan & Toal, 2006).

Flat-panel glass windows are commonly used for underwater camera housings. While domes provide optical advantages, they have to be specially engineered to fit the camera and the integration is not trivial. Flat pane windows are hence simply a much less expensive and more flexible choice. On the other hand, flat ports introduce significant distortions due to the refraction at the air-glas and glas-water interfaces. The predominant way to handle the distortions is to use a standard perspective projection model and to perform a standard camera calibration in-situ, i.e., in the water or by including estimated correction factors, see e.g., (Shortis & Harvey, 1998; Gracias & Santos-Victor, 2000; Pessel et al., 2003; Pizarro et al., 2003; Lavest et al., 2003; Negahdaripour et al., 2006, 2007; Brandou et al., 2007; Sedlazeck et al., 2009; Johnson-Roberson et al., 2010; Kunz & Singh, 2010; Beall et al., 2011; Kang et al., 2012).

But Treibitz et al. (2008, 2012) show that flat port cameras do not possess a single viewpoint (SVP), i.e., the perspective projection model is invalid for flat ports. This is also supported by other works (Li et al., 1997; Kunz & Singh, 2008; Chari & Sturm, 2009; Gedge et al., 2011; Yamashita et al., 2011; Sedlazeck & Koch, 2011; Jordt-Sedlazeck & Koch, 2012; Agrawal et al., 2012; Servos et al.,

2013; Jordt-Sedlazeck & Koch, 2013; Chen & Yang, 2014; Jordt-Sedlazeck &
30 Koch, 2012; Yau et al., 2013).

In Kunz & Singh (2008) the errors caused by not compensating the refractive distortions are discussed in some detail and they are identified to be significant, however no solution to this problem is presented. A mathematical model of underwater imaging through planar glass ports is introduced in (Chari & Sturm,
35 2009). Matrices corresponding to fundamental and homography matrices are derived. They however depend on the incident angle of the light ray corresponding to each image pixel, so they can not be used directly for underwater vision methods. Since no continuation of this work was published, their results remain as theoretical considerations of conceptual value. In addition to a deep
40 theoretical treatment of the general problem, Treibitz et al. (2008, 2012) provide an approach for a single refractive layer, i.e., when the window is negligibly thin and the problem can be reduced to only a single air-water interface.

Important insights into the problem and ways towards a solution are presented in (Agrawal et al., 2012) where a flat port camera is identified to be in
45 fact an axial camera. Agrawal et al. (2012) derive a 12th degree polynomial that must be solved to project a 3D point onto an image plane in this case. A method is proposed for calibration of the camera but it requires knowledge of the full 3D geometry of the calibration points in the environment - a requirement which is difficult if not impossible to fulfill in underwater applications. Furthermore,
50 the underlying axial model does not allow for a rectification of single images as the axial model implies that the points are lying on complex curves. Correspondences across multiple images can be in principle be exploited, but this is computationally very complex as also pointed out in Jordt-Sedlazeck & Koch (2013).

55 When using multiview methods, the SVP model can lead to reasonable results as explicitly discussed in (Kang et al., 2012). Nevertheless, Jordt-Sedlazeck & Koch (2013) build on the results from Agrawal et al. (2012) by proposing a refractive Structure from Motion (SfM) method by augmenting the standard perspective SfM process by incorporating a new error function in the optimiza-

tion and report clear improvements. While this is an interesting approach, it requires sufficiently many images with sufficiently different views of the scene and it is still computationally very demanding. Note that though we use stereo vision for validation purposes in the experiment section, we do not use any two- or multiview information. Stereo data is just used in our experiments as it facilitates a metric analysis of the errors. Our method is perfectly suited for the calibration and rectification of single camera images, i.e., the most general case of underwater vision. Our method can hence also be of interest for stereo or multiview approaches - including refractive ones like in (Jordt-Sedlazeck & Koch, 2013) - by providing excellent initial guesses for the camera parameters, hence allowing for faster convergence and possibly even more accurate results.

In this article we make two main contributions. First, we discuss the problem of underwater camera modeling from a practitioners viewpoint. We provide illustrating examples of the underlying effects and their relevance to real world applications. To some extent, this also bridges some apparent contradictions found in the literature that can be explained when contrasting theoretical considerations with typical application cases. Second, we provide an approach for calibration and refraction correction of underwater images that is very convenient to use in real world applications and that is at the same time very accurate. This *pinax* model is based on a virtual *pinhole* camera model - for which we show that it is applicable for real world underwater housings where the camera is relatively close to the flat-pane - while using the projection function of an *axial* camera. The *pinax* model incorporates the water refraction index, for which - as also experiments show - it is sufficient to derive it through (estimated) salinity to achieve accurate results. It is hence sufficient to calibrate the underwater camera only once in air, thus replacing tedious in water calibrations before or during missions. For the rectification, a look-up table is generated using as mentioned the projection function of the axial model, for which we show that it can be used in a significantly simplified fashion within the *pinax* model. The look-up table can be easily computed a priori and allows very fast real-time refraction correction of single images. Real world experiments

with different cameras in different sweet and salt water environments show that the pinax model outperforms standard methods. The code for using the pinax model is provided in MATLAB, and the Robot Operating System (ROS) and freely usable for academic purposes.

95 2. Problem Formulation and Motivational Examples

2.1. Flat-Panel Camera Setup

The following setup is considered. A physical camera pCam that follows the standard SVP model with an intrinsics matrix ${}^p\mathbf{K}$ is enclosed in a water sealed housing with a flat glass panel through which it observes the underwater
 100 environment. The glass panel is flat and both sides are parallel. The glass panel introduces distortions that are to be handled by a virtual camera model vCam that interprets the environment scene from the physical camera pCam . The overall underwater setup of the physical camera plus its housing with a flat-pane window submerged into water is denoted as the underwater camera
 105 uCam . When the underwater camera is in air, e.g., for the calibration, it is denoted with aCam .

If not mentioned otherwise, we refer in the following with the term *camera* to the complete underwater set-up and use the terms *virtual camera* and *physical camera* to refer to the model vCam of the glas-panel refraction, respectively to
 110 the in-air physical device pCam inside the housing.

The main object of interest for this article is the virtual camera model vCam to handle the refraction induced distortions. The related notations and a schematic view are presented in Fig. 1. Following parameters are used:

- d_0 - distance from the center of projection of pCam to the glass window,
- 115 • d_1 - thickness of the glass,
- x - distance to point of intersection of the light ray with the camera axis,
- Δx - length of the focus section,

- n_a, n_g, n_w - refraction indexes (scaled so that $n_a = 1$),
- \mathbf{n} - normal vector to the glass surface,
- α - incident angle.

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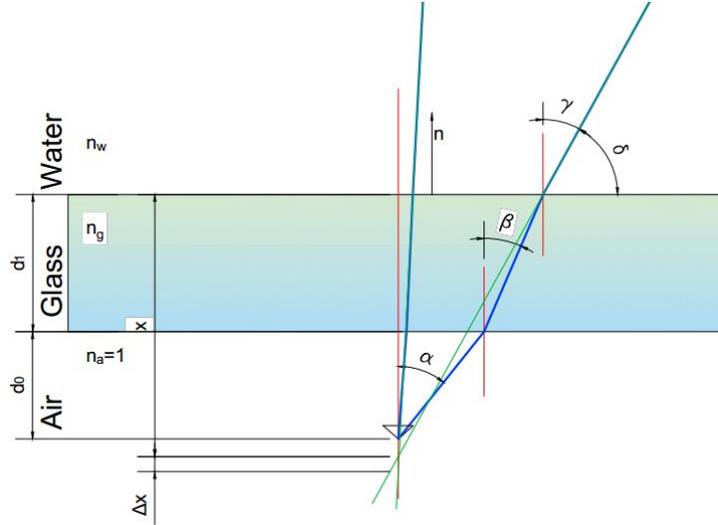


Figure 1: Schematic view of a Flat Port setup: d_0 - distance from the center of projection to the glass window, d_1 - thickness of the glass, x - distance to point of intersection of the light ray with the camera axis, Δx - length of the focus section, n_a, n_g, n_w - refraction indexes, scaled so that $n_a = 1$, \mathbf{n} - normal vector to the glass surface, α - incident angle. The blue line represents the physically accurate light ray; the green line is the apparent ray traced back to the camera's optical axis.

2.2. The Flat Port Setup as an Axial Camera

As shown in (Agrawal et al., 2012), the physically accurate model of a flat-port underwater camera corresponds to an axial camera model. So, light rays creating the image do not intersect in one point, as in the SVP pinhole model, but they all cross one line, called the axis of the camera. Using the pinhole camera model requires therefore to approximate the focus section, i.e., the line segment on the axis on which rays cross, with a single point. The conclusion is that the quality of this approximation depends directly on the length Δx

125

of this section. In the limit case, the pinhole camera can be seen as an axial
 130 camera where the focus section of the camera axis is infinitely short. To analyze
 the refraction, ray tracing through the air-glass-water interface and the appar-
 ent intersection of the rays in the water with the camera axis can be modeled
 (compare Fig. 1):

$$\begin{aligned}
 \beta &= \arcsin \frac{\sin \alpha}{n_g} \\
 \gamma &= \arcsin \frac{\sin \alpha}{n_w} \\
 \delta &= \frac{\pi}{2} - \gamma
 \end{aligned}$$
 135

For the sake of simplicity, we assume that the refractive plane normal and
 therefore the camera axis in the axial model is parallel to the optical axis of the
 camera. This assumption is without loss of generality since the incident angle
 140 α , i.e., the only parameter related to camera rotation, is one of the inputs,
 which can be easily rotated by a fixed off-set. For the sake of completeness, the
 equations for finding incident angles α given the camera pose in the housing is:

$$\begin{aligned}
 \mathbf{v}_0 &= \mathbf{K}^{-1} \mathbf{p} \\
 \alpha &= \arccos \frac{\mathbf{v}_0^T \mathbf{n}}{|\mathbf{v}_0| |\mathbf{n}|}
 \end{aligned}$$

145 where \mathbf{K} is the intrinsic parameter matrix and \mathbf{p} represents pixel coordinates
 on the image.

The focus distance x for each light ray (Fig. 1) can be computed as:

$$x = \tan \delta (d_0 \tan \alpha + d_1 \tan \beta)$$

2.3. Length of the Focus Section

150 Consider an example setup with a glass refraction index $n_g = 1.5$, a water
 refraction index of $n_w = 1.335$ and a glass thickness of $d_1 = 10mm$. Plotting
 the change of x as a function of the incident angle α and of the distance d_0
 illustrates a very important aspect (Fig. 2). As d_0 grows, the changes in the
 focus distance depending on the incident angle α (along X axis) become more
 155 significant, i.e., there is a higher range of focus distances with increasing d_0 .
 This is further illustrated in the following.

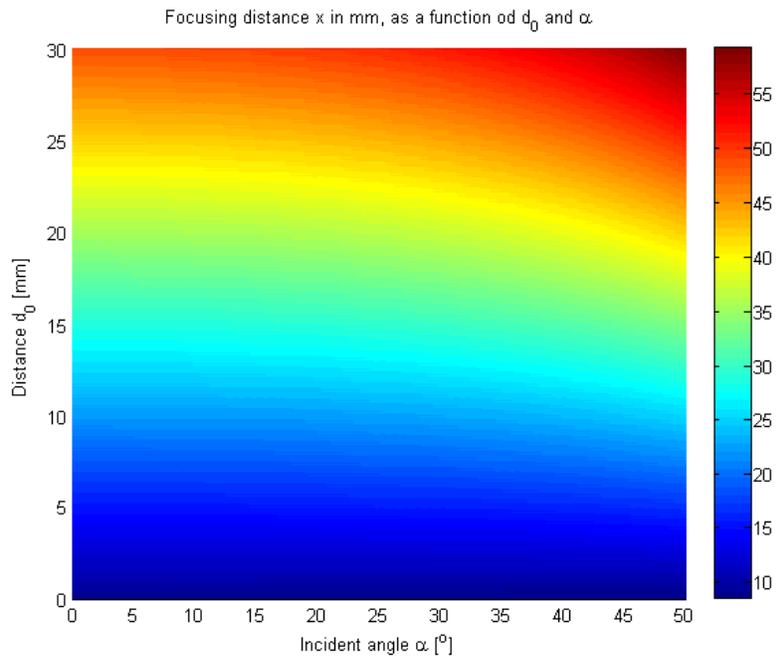


Figure 2: The focus distance x (in mm) as a function of d_0 and α for an example setup with a glass refraction index of $n_g = 1.5$, a water refraction index of $n_w = 1.335$ and a glass thickness of $d_1 = 10\text{mm}$. It can be seen that the changes in the focus distance x for different incident angles α become more significant for increasing distances d_0 of the physical camera to the flat-pane.

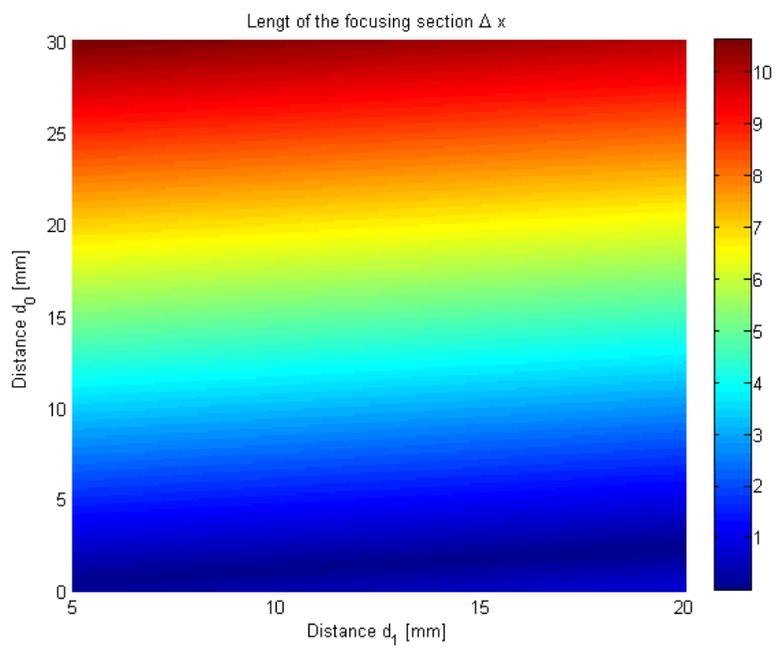


Figure 3: The length of the focus section (Δx) as a function of d_0 and d_1 . It can be seen that the influence of the distance d_0 of the physical camera to the flat-pane is more significant than the thickness d_1 of the glas-pane.

Fig. 3 shows the length of the focus section Δx as a function of d_0 and d_1 . We assume in this illustrative example a glass refraction index of $n_g = 1.5$ and a water refraction index of $n_w = 1.335$. The plot shows that changes in d_0 are much more significant than changes in d_1 , i.e., the distance of the camera to the flat-pane window has a stronger effect than the thickness of the glass window. This effect is caused by a relatively small difference between the refraction index of glass (≈ 1.5) and the average water refraction index (≈ 1.33 , (Roswell et al., 1976)) compared to the more significant refraction on the glass-air interface. In Fig. 3 it can be also observed that the best approximation of the axial camera model with a pinhole model occurs for small values of d_0 .

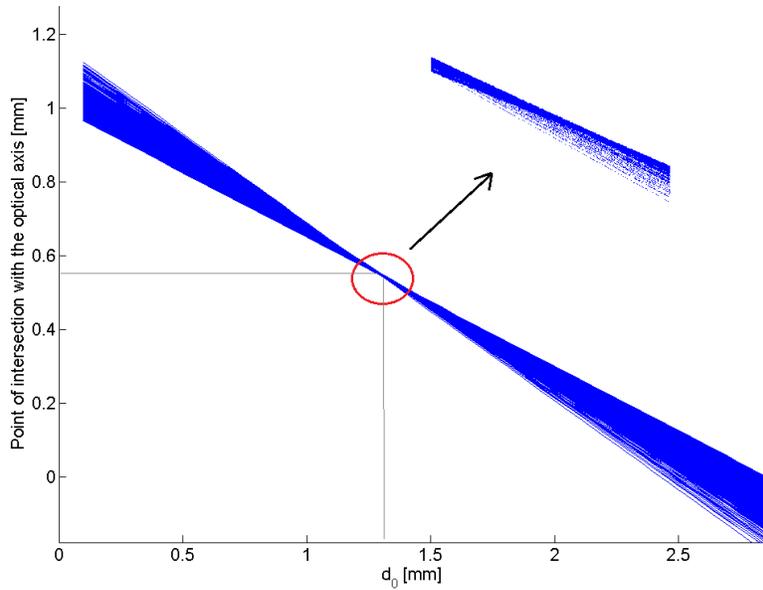


Figure 4: Example distances where the light rays traced back from the water cross the optical axis of the camera depending on d_0 . Different lines correspond to different incident angles ranging from 0 to 35 degrees, i.e., a physical camera with a field of view of 70 degrees.

To further motivate and illustrate this, Fig. 4 shows where the light rays in water cross the camera’s optical axis for different values of d_0 . Each line on the graph corresponds to a different incident angle ranging from 0 to 35 degrees,

170 i.e., a physical camera with a field of view of 70 degrees. It can be seen that
they never cross the same spot, but for some optimal d_0 , they are very close to
intersecting in one point. To find this optimal value for some given parameters
the following method is used. We implemented ray tracing based on the above
formulation of the model. Then non-linear optimization is used to minimize
175 the length of the section where light rays back-traced from the water intersect
with the camera optical axis. For example, for the case where $d_1 = 10mm$, the
glass refraction index $n_g = 1.5$, and the water refraction index $n_w = 1.335$, the
method converges to $d_0 = 1.4282mm$ where all light rays intersect the optical
axis on a section Δx that is only $0.0079mm$ long, i.e., within a very good
180 approximation of a single point. The result of this numerical analysis allows to
define the middle of this section Δx as a secondary center of projection placed
 $0.5851mm$ away from the glass panel. For this case, the virtual camera can be
treated as an SVP camera and represented with the pinhole model.

This example motivates that although the pinhole camera model does not
185 represent the actual physical state, for the purpose of underwater vision it may
be used as basis for a model if the distance between the center of projection of
 pCam and the glass plane is very small. This is a realistic assumption as there
are no reasons to design excessive housing sizes, i.e., the physical camera inside
a housing is usually placed quite close to the flat-pane window.

190 2.4. Influence of the Distance of the Camera to the Flat-Panel

The influence of the distance of the physical camera to the flat pane is
now further illustrated in a other motivational example. The thickness of the
glass panel d_1 is assumed to be constant at $d_1 = 10mm$. For different values
of d_0 , the camera is calibrated with a standard procedure. The calibration
195 input data is based on 27 simulated checkerboards in 3D space (Fig. 5) by
projecting the corner points to the image plane using the full physical model
including refraction. This forward projection requires solving the twelve-degree
polynomial introduced in (Agrawal et al., 2012). This data is hence used to
calibrate the camera as if it would be underwater.

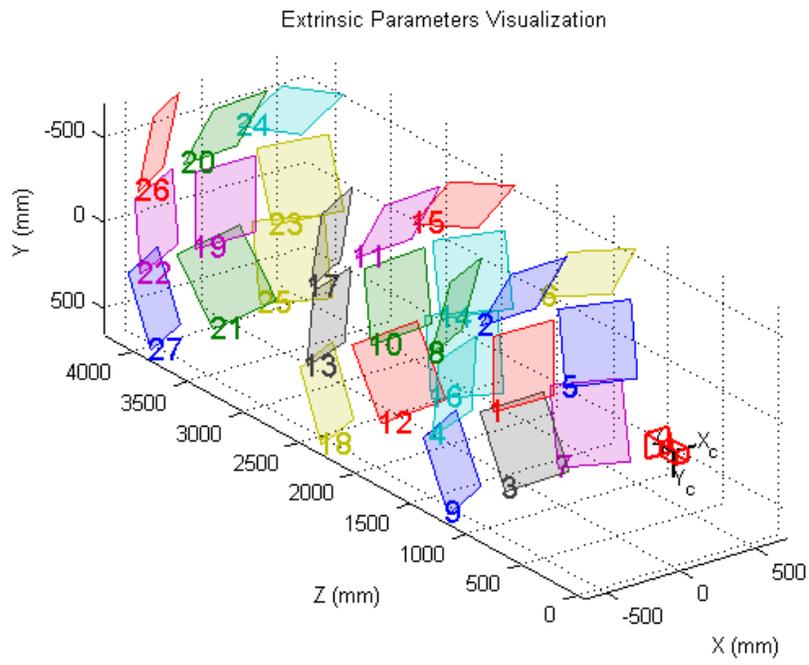


Figure 5: Poses of the calibration patterns used for a further motivational experiment in simulation.

200 In a second step, a set of 100 random 3D points in front of the underwater camera is generated. This set is then projected onto the image plane twice for each d_0 . Once, the projection is performed with the full physical model to get the expected image coordinates and a second time with the camera matrix from calibration using the pinhole model including undistortion. Then the distance
205 between corresponding points (also called reprojection error) is calculated and used to evaluate the pinhole approximation for the the different d_0 . Fig. 6 and 7 show the results by plotting the average distance between corresponding points for the different d_0 values.

Note that we consider here values of d_0 up to $500mm$. Values within this
210 range can be found in the literature for experimental setups, especially in the highly relevant works of Agrawal et al. (2012) and Treibitz et al. (2012). Such big values are used because then the effects of the axial camera model are clearly visible and for example the position of the camera in the housing can be found with nonlinear optimization. There may be applications were the distance of the
215 physical camera to the flat-pane is quite large, e.g., when observing objects in an aquarium and the physical camera needs to keep a significant clearance to the aquarium window for some reason. However, this scenario is very unrealistic for underwater cameras. Excessive housing sizes to allow for significant distances d_0 are neither necessary nor desirable for underwater applications.

220 2.5. Rectification Accuracy near the Calibration Distance for SVP

The following simulation example is designed to illustrate that regardless of the setup parameters it is possible to get a reasonable approximation of the physical state with a standard SVP pinhole model, if the observed part of the scene is always observed from roughly the same distance D and the camera
225 calibration was done at about the same distance, i.e., the calibration pattern was moved underwater in front of the camera in also roughly the distance D . This illustrates that, e.g., for mosaicking with a vehicle camera in a (roughly) fixed distance over ground, good rectification results with a pinhole model can be achieved if the calibration pattern was moved in water at roughly the same

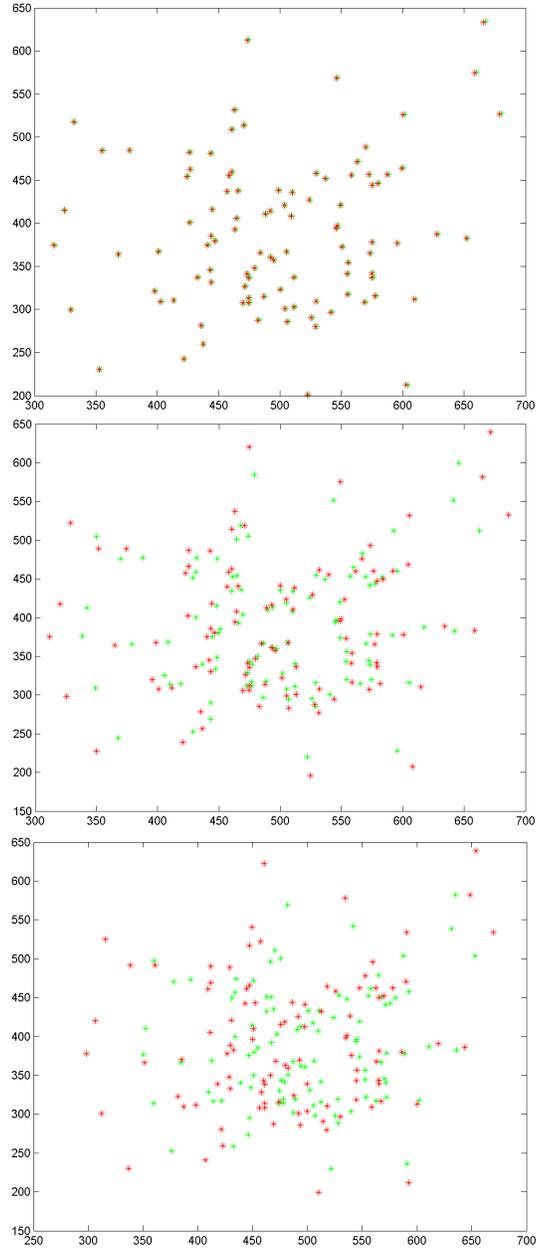


Figure 6: An example with a refraction-based (green) and a pinhole (red) projection of random points in the scene for $d_0 = 1mm$ (top), $300mm$ (center), and $500mm$ (bottom) respectively. Note the increasing deviations in the models with increasing d_0 .

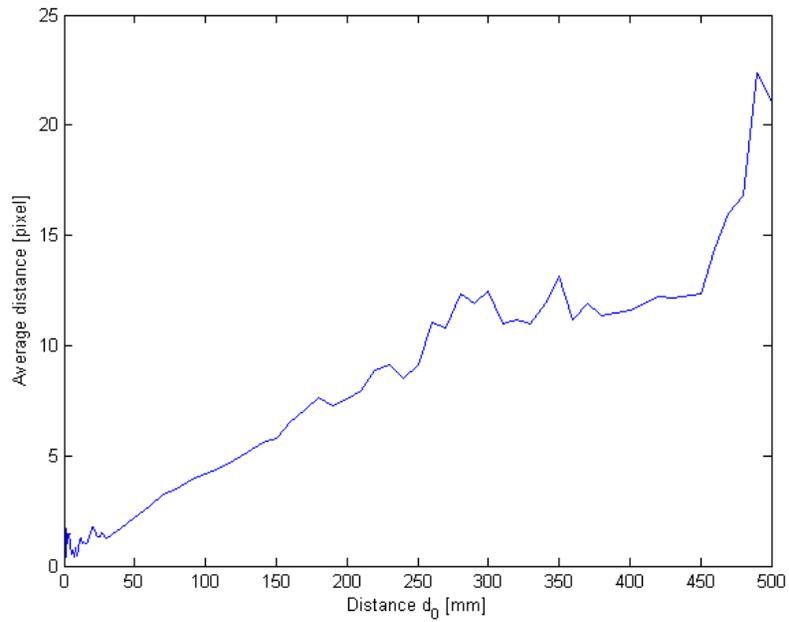


Figure 7: An example of the reprojection error for changing d_0 . Note that for $d_0 = 0 - 10\text{mm}$ the errors caused by the SVP approximation can be neglected. The graph is not as smooth as may be expected, e.g., as in Fig. 3, because the simulated patterns were not always in the optimal positions for calibration, e.g., they did not always cover the whole field of view of the camera - which is a very natural effect that can also be observed in real world conditions.

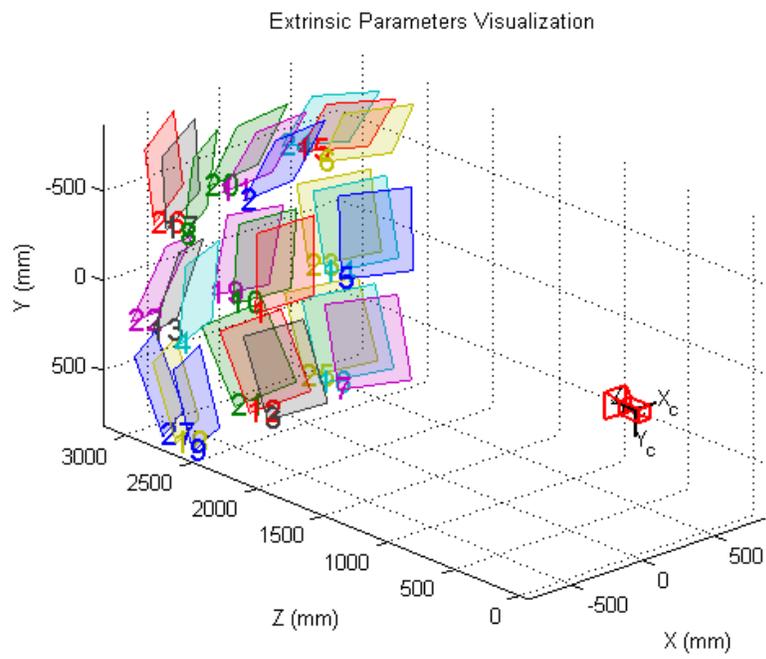


Figure 8: Arrangement of calibration patterns used for calibrating the camera in an example illustrating the effects of the distance of the calibration in the pinhole model under unfavourable parameter conditions.

230 distance. On the other hand, errors emerge once the camera is looking at parts
of the scene that are closer or further then D .

The setup parameter are as follows: $d_0 = 80mm$, $d_1 = 20mm$, i.e., a significant amount of space between the focal point of the physical camera lens and the glas panel plus a relatively thick glas pane. So, the parameters, especially d_0 ,
235 are in this case relatively unfavourable. In this illustrating example, the camera is calibrated with 27 pattern poses spread around a point 2 m away from the camera (Fig. 8). Then test points are generated again randomly but around a given distance from the camera and, using the same method as above, projected onto the image plane. The reprojection error against the point distance to the
240 camera is shown in Fig. 9.

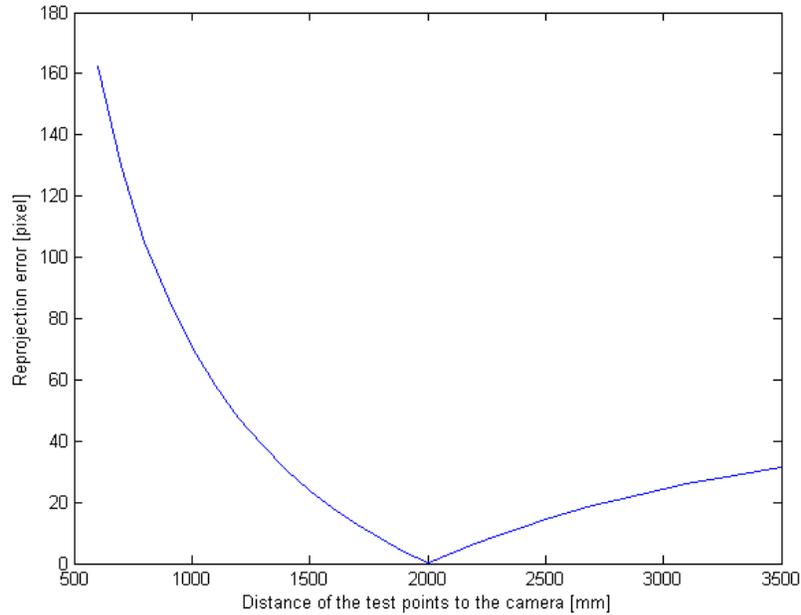


Figure 9: The reprojection error for a changing distance of observed points to the camera as an example that the SVP pinhole model performs well if the observed points are close to the distance in which the camera was calibrated with an SVP model. In this simulation example, the camera was calibrated with patterns around 2 m away from the camera (Fig.8), which is exactly the distance were the reprojection error is minimal.

The pinhole model holds very well only around the distance of calibration. This shows that for some specific applications, where minimizing d_0 is not possible, e.g., due to physical size of the lens as part of the camera subcomponent in the housing, the pinhole model can still be effectively used if the environment
245 is observed from a known constant distance. As mentioned, one of the applications fulfilling this assumption can be seabed mosaicking with constant altitude control of the observing AUV.

This effect can also be observed in (Kang et al., 2012) where the quality of Structure from Motion under an SVP model is investigated and good results are
250 reported even for a larger distance of the camera to the window. The camera rig used in the experiments leads to a constant distance between the camera and the investigated object, hence the effect illustrated in this subsection takes place.

3. The Pinax Model

255 3.1. Overview

Based on the previous considerations, we propose a system where a few setup assumptions are used to compensate for the refraction-based distortions of the image. Specifically, a transformation is computed to undistort and rectify the camera images. The resulting images can be directly used for example in stereo
260 vision algorithms or for mosaicking to just name two examples.

The following assumptions are made:

1. The distance d_0 between the glass and the center of projection is small and near the optimal spot d_0^* where the rays traced back from the water cross in a minimum focus section Δx^* .
- 265 2. The optical axis of the camera is perpendicular to the glass surface. To achieve this a correcting transformation may be applied, e.g. for converged stereo systems.
3. The glass thickness and its approximate refraction index are known, e.g., using standard refraction indexes for glass or plexiglass.

270 4. The water refraction index is approximately known from tables, e.g., from
(Roswell et al., 1976).

Fulfilling these assumptions allows to assume a pinhole model for the virtual camera and hence allows to model the refraction-based distortions very efficiently. It also makes it possible to omit any underwater calibration procedures.
275 The first assumption in the above list is of course by far the strongest and most significant one. As motivated before, it is at least not unrealistic to assume that underwater housings are minimized for size and that hence the physical camera inside the housing is placed as closely as possible to the window. This assumption is also supported by the real world experiments presented later on.

280 3.2. In-Air Calibration

As a first step in our method, the physical camera pCam is calibrated once in air, i.e., its intrinsic matrix ${}^p\mathbf{K}$ is determined using any standard calibration process (Hartley & Zisserman, 2003). From a practical viewpoint it is very interesting to note that the front window does not have to be removed from the
285 housing. Concretely, the physical camera pCam can be calibrated by calibrating the underwater camera uCam in air, i.e., by determining its intrinsic matrix ${}^u\mathbf{K}$.

The air-glass and glass-air refractions only lead to a change in scale in the images, which is part of the extrinsics, and the relative geometric relations between points in the scene are preserved. A calibration process of the underwater
290 camera uCam in air is hence the same as if calibrating pCam , i.e., ${}^p\mathbf{K} = {}^u\mathbf{K}$.

If the calibration of the physical camera was already done outside of the housing, e.g., by the manufacturer, it is of course perfectly fine to use that data. The in-air calibration of the full underwater system is only an option that is very convenient to use for already existing complete camera systems. For high
295 quality in-air calibration, the according tool in CamOdoCal(Heng et al. (2013, 2014, 2015)) is used in our experiments presented later on in Sec. 4.

Table 1: Optimal $d_0^*/{}^v d_0^*$ of the centers of projection of the physical/virtual camera for different glass thicknesses and two common salinity values

d_1 [mm]	$n_w = 1.333$ (sweet water)	$n_w = 1.342$ (salty water)
1	0.15mm/0.06mm	0.14mm/0.06mm
3	0.45mm/0.18mm	0.42mm/0.17mm
5	0.76mm/0.31mm	0.70mm/0.29mm
10	1.52mm/0.61mm	1.40mm/0.58mm
15	2.28mm/0.92mm	2.10mm/0.87mm
20	3.04mm/1.22mm	2.80mm/1.15mm

3.3. Determining the Optimal d_0^*

In an ideal scenario, the optimal distance d_0^* between the glass and the center of projection can be taken into account when designing a new underwater camera. More precisely, the optimal distance ${}^p d_0^*$ of the physical camera should be taken into account as the model of the virtual camera ${}^v Cam$ has its own, slightly different ${}^v d_0^*$ as discussed in more detail in the following section.

As already sketched in section 2.3, ray tracing and non-linear optimization can be used to minimize the length of the section where light rays back-traced from the water intersect with the camera optical axis. The MATLAB code for these computations is provided as supplementary material to this article.

Using ${}^p d_0^*$ in a camera design is as mentioned the ideal scenario and we only include its computation here for the sake of completeness. In most application cases, the underwater camera is an off-the-shelf system or an already finished design, respectively other design constraints on the housing or the physical camera/lens components may apply. But we consider it safe to assume that for any typical underwater housing the real distance ${}^p \hat{d}_0$ is sufficiently close to ${}^p d_0^*$.

As illustrated in Tab. 1, ${}^p d_0^*$ tends to be in the order of a few millimeters and less. At the same time, the physical length of lenses tends to be in the order of their focal lengths, i.e., the center of projection tends to be at the front-end of the lenses of the camera device. Hence placing the physical camera as close as

possible to the glass-pane with maybe a small air gap, i.e., using the standard default set-up for typical underwater cameras, leads to a close approximation of ${}^p d_0^*$ by ${}^p \hat{d}_0$ with negligible errors. An exact quantification of the related errors is discussed below in the following sections.

3.4. Refraction and Lens distortion Modeling with Maps

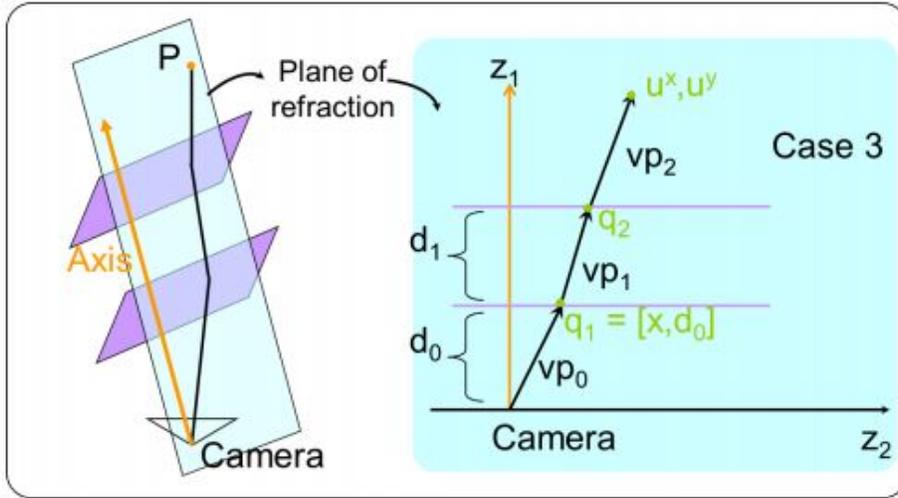


Figure 10: Parameters of the analytical forward projection through a flat refractive panel with a 12th degree polynomial (From supplementary materials to Agrawal et al. (2012)).

The main conclusion from the assumptions in Sec. 3.1, especially from assumption 1 about the distance of the physical camera to the window, is that a pinhole camera model can be used for the virtual camera model with a negligible error. Concretely, we exploit this insight by defining a virtual pinax plane $\mathbf{p}_{\mathbf{pa}} = (d_{pa}, \mathbf{n}_{\mathbf{pa}})$ that is assumed to be at distance d_{pa} in the scene with a normal vector $\mathbf{n}_{\mathbf{pa}}$ that is parallel to the camera axis. The distance d_{pa} is set fixed to $5m$ as this is considered a typical viewing distance; but as discussed below, the exact value is of minor interest as points on pinax planes at different distances behave similar due to the virtual pinhole camera property that follows from small values of d_0 . $\mathbf{v}_{\mathbf{p}}$ and $\mathbf{p}_{\mathbf{p}}$ denote homogeneous image pixel coordinates. The intensity or color value of a given pixel is denoted as $I(\mathbf{p})$.

Each point \mathbf{v}_p from the image vI of the virtual camera vCam is projected onto the pinax plane \mathbf{p}_{pa} using a pinhole camera projection. Then this point \mathbf{m}_w is projected forward to the inside surface of the glass panel (point \mathbf{m}_a) using the method derived from Agrawal et al. (2012). Now \mathbf{m}_a may be transformed to \mathbf{P}_p with the in air calibration parameters of pCam to obtain pixel coordinates in the distorted image pI . This last step may be performed using any camera and lens distortion model, referenced in Algorithm 1 with the subroutine *project3dToImage()*. When the calibration of the physical camera is based on a pinhole camera model with no lens distortion, this is:

$$\begin{aligned} {}^p p = {}^p K \cdot \mathbf{m}_a \\ {}^p p = {}^p p \cdot \frac{1}{{}^p p_z} \end{aligned}$$

In order to find the point \mathbf{q}_1 (point corresponding to \mathbf{m}_a , expressed in coordinate frame z_1, z_2) as shown in figure 10 (compare also figure 1), the twelfth-degree polynomial method derived in Agrawal et al. (2012) is used. For the sake of completeness we shortly present the most important findings of Agrawal et al. (2012) here. As discussed before, it can be shown that a camera behind a flat glass panel is an axial camera. The camera axis is assumed to be identical to the optical axis. When tracing the light path all the refractions happen in one plane, called plane of refraction (POR), so the analysis can be conducted in 2D. To do this \mathbf{m}_w must be projected to POR. The new coordinate system is defined as follows. Axis z_1 is identical to the camera axis, z_2 is orthogonal to z_1 and lays on POR. This way \mathbf{m}_w projected to $[u^x, u^y]$ may be used for calculations.

$$(k_1^2 D_1 + k_3^2 D_2 - k_2^2 D_1 D_2)^2 - 4k_1^2 k_3^2 D_1 D_2 = 0$$

where

$$k_1 = x(d_0 + d_1 - u^y)$$

$$k_2 = (u^x - x)$$

$$k_3 = -d_1 x$$

$$D_1 = d_0^2 n_g^2 + n_g^2 x^2 - x^2$$

$$D_2 = d_0^2 n_w^2 + n_w^2 x^2 - x^2$$

```

let  $M$  be an associative array
for  ${}^v \mathbf{p} \in {}^v I$  do
   $\mathbf{m}_w = {}^v K^{-1} \cdot {}^v \mathbf{p} \cdot d_{pa}$ 
   $z_1 = (0, 0, 1)^T$ 
   $z_2 = z_1 \times (z_1 \times \mathbf{m}_w)$ 
   $u^x = z_2 \cdot \mathbf{m}_w$ 
   $u^y = z_1 \cdot \mathbf{m}_w$ 
   $q_1 = \text{solve12thDegPoly}(\text{setupParams}, [u^x, u^y])$ 
   $\mathbf{m}_a = q_{1.x} \cdot z_2 + q_{1.d} \cdot z_2$ 
   ${}^p \mathbf{p} = \text{project3dToImage}(\text{CameraAndLensModel}, \mathbf{m}_a)$ 
  store key-value pair  $({}^v \mathbf{p}, {}^p \mathbf{p})$  in  $M$ 
end

```

Algorithm 1: Creating correction maps in the Pinax Model (compare Fig. 11).

The method solving this polynomial to find q_1 is referenced with the subroutine *solve12thDegPoly()* in Algorithm 1.

This procedure that combines a pinhole forward and an axial backward projection has to be computed only once and leads to an image transformation for undistortion and rectification stored in a lookup table (compare Algorithm 1, 2 and Fig. 11). The main contribution in the context of the pinax model is of course Algorithm 1, i.e., the way the correction map is created, while Algorithm 2 is just the standard procedure for applying correction maps for rectification, which is included here for the sake of completeness.

```

Let  $M$  be an associative container created with algorithm 1
for  ${}^v\mathbf{p} \in {}^vI$  do
  look up value  ${}^p\mathbf{p}$  for key  ${}^v\mathbf{p}$  in  $M$ 
   $\mathbf{a} = \text{floor}({}^p\mathbf{p})$ 
   $b_x = {}^p\mathbf{p}_x - \mathbf{a}_x$ 
   $b_y = {}^p\mathbf{p}_y - \mathbf{a}_y$ 
   $c_1 = b_x \cdot {}^pI(\mathbf{a}) + (1 - b_x) \cdot {}^pI(\mathbf{a} + (1, 0, 0)^T)$ 
   $c_2 = b_x \cdot {}^pI(\mathbf{a} + (0, 1, 0)^T) + (1 - b_x) \cdot {}^pI(\mathbf{a} + (1, 1, 0)^T)$ 
   ${}^vI({}^v\mathbf{p}) = b_y \cdot c_1 + (1 - b_y) \cdot c_2$ 
end

```

Algorithm 2: Applying Pinax correction maps

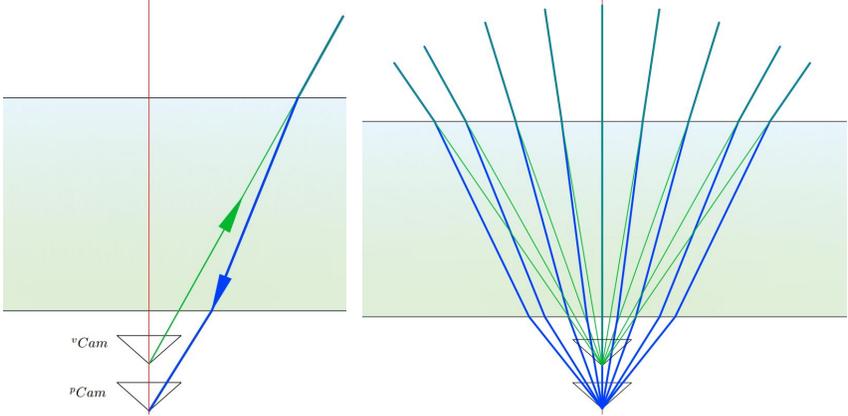


Figure 11: Left: The map creation in the Pinax model that combines a projection from the virtual pinhole camera to the pinax plane (green ray) and back with an axial projection to the physical camera (blue ray). Right: The virtual (green) rays are good approximations of the physical rays (blue) once they cross from the glass panel into water - and the small d_0 assumption is fulfilled.

3.5. Numerical Error Analysis

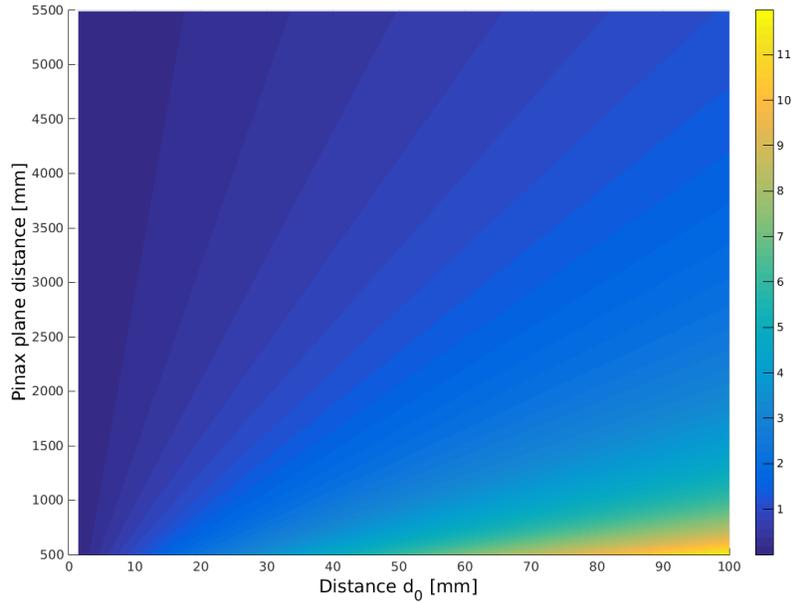


Figure 12: Errors between the look up pixel value for the optimal $*d_0$ and the pinax plane distance of 5m and scene points that are at different distances than the pinax plane, respectively for which in addition d_0 deviates from the optimal $*d_0$. Note that as long d_0 is close to $*d_0$, the location of the point in the scene has no influence.

The essential assumption in our model is that the correction computed for points in the pinax plane also generalizes for other points in the scene. Furthermore, we postulate that minor variations in \hat{d}_0 are negligible and that typical
 370 underwater cameras are already designed such that near optimal conditions are fulfilled. Fig. 12 shows the maximum errors between look up pixel values for the optimal $*d_0$ and a pinax plane distance of 5m and scene points that are at different distances than the pinax plane, respectively if in addition d_0 deviates from the optimal $*d_0$.

375 Fig. 12 shows that the errors are very small, i.e., in the order of at most a few millimeters over some meters distances, even with significant deviations of d_0 from $*d_0$ of up to 40mm, i.e., under the presence of severe air gaps between the

camera and the front panel. Only if the physical camera is significantly placed away from the glass panel pronounced errors occur. If d_0 is quite close d_0 ,
 380 i.e., if the air gap is small, the theoretical errors are even negligible considering realistic camera parameters. It can also be noted that the error becomes smaller for larger distances of the scene points.

3.6. The Role of Changes in the Water Refraction Index

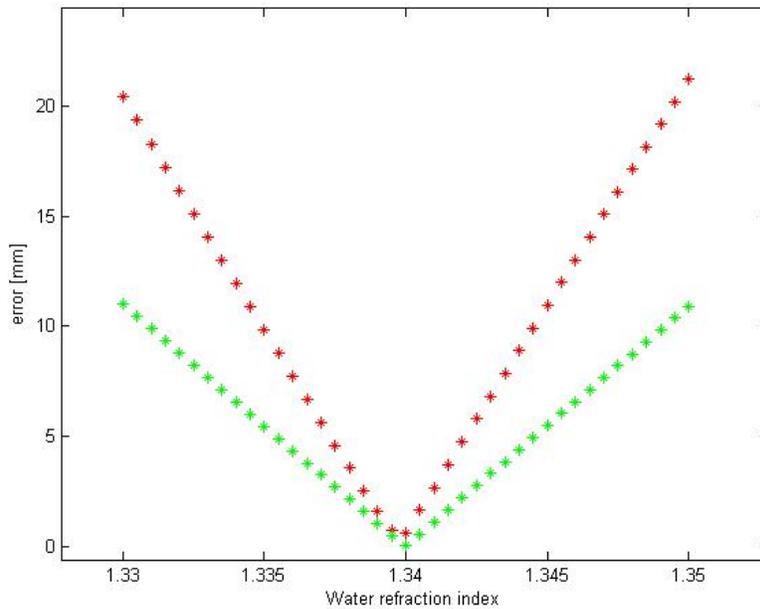


Figure 13: The maximum (red) and average (green) error caused by changes of the water refraction index in an example camera set-up. The error is measured as the displacement of the estimated light ray from the proper one at a 5 m distance from the camera.

The computation of the pinax correction map takes the refraction index
 385 of the water into account. It is important to note that even seemingly small changes from a nominal value of $n_w = 1.34$ have noticeable effects. Consider our standard example set-up in combination with the water refraction indexes $n_w = 1.33$ and $n_w = 1.35$, i.e., only about $\pm 0.75\%$ from the nominal value. Fig. 13 shows the errors for the different refraction indexes in this example.

390 The errors are computed as the displacement of the estimated light ray from
the proper one at a 5 m distance from the camera. Both the maximum error,
which occurs for rays under the maximum incident angle, as well as the average
error over all rays are shown. The errors are substantial, i.e., though there is
only a very small change in the water refraction index it is very beneficial to
395 take it into account. This also holds for other methods in general as shown in
the experimental results section. While it is sufficient to simply recompute the
correction map in the pinax model, which can be done very fast and without the
need of gathering any additional vision data, the standard in water calibration
approach requires a new recording of in-situ data to avoid errors.

400 In our experiments presented below, we simply use estimated salinity and
the related refraction indexes from tables (Roswell et al., 1976), which we found
sufficiently precise to accommodate for the effects of changing water refraction
indexes. Nevertheless, the exact water refraction index can also be computed
from physical parameters, e.g., by using the formulas from Millard & Seaver
405 (1990) or Quan & Fry (1995). The predominant factors is the influence of
salinity followed by a much lesser extent by temperature. Very commonly used
CTD sensors provide exactly this information, i.e., it is very simple to get an
exact indirect measurement of the water refraction index if needed. As the
computation of the correction map is relatively fast (see Sec.4.1), this even
410 allows for a online re-computation of the correction map during the mission if
the conditions change, e.g., if the camera on a vehicle operating at sea passes an
sweet water inflow or if a mission ranges from warmer shallow waters to much
colder deep waters.

4. Experiments and Results

415 4.1. Overview

In this section, we complement the previous theoretical discussions and nu-
merical analyses of the pinax model with quantitative evaluations with real
underwater camera systems. The underwater systems are based on various

components from different third parties. None of the systems or system compo-
420 nents was designed with knowledge of the pinax model. We use stereo cameras
here for the only reason that they allow for an easier qualitative and quanti-
tative evaluation of the errors in the rectification. The pinax calibration and
rectification itself is of course directly applicable to single monocular cameras
and does not use any stereo- or multi-view information.

425 First, run-times for generating and applying the refraction correction maps
are presented in Sec. 4.2. Especially the use of a correction map for rectifica-
tion is extremely fast and can hence be applied in real-time on a video stream.
Second, qualitative results from field work where the pinax model is used for
in-air calibration are presented in Sec. 4.3. The qualitative results are based
430 on several third party systems including a custom-made underwater camera on
the Ifremer vehicle Vortex Brignone et al. (2011) and several COTS cameras in
custom-made underwater housings, e.g., on the AUV Sparus Mallios et al. (2011)
from the University of Girona and the AUV Seacat Enchev et al. (2015) from
ATLAS Elektronik. Third, quantitative evaluations are presented in Sec. 4.4
435 where the pinax model is compared to state of the art underwater calibration.
The experiments are conducted with a Bumblebee XB3 with dual stereo, i.e.,
three monocular cameras at two different baselines, in a custom-made underwa-
ter housing and with a GoPro Hero3+ Black Edition stereo rig in a consumer
housing from GoPro. The accuracy of underwater stereo computations on arti-
440 ficial checker-board patterns is used in the quantitative evaluations as a metric
of rectification accuracy.

4.2. Run-Times for Generating and Applying the Refraction Correction Maps

One of the strengths of the pinax model is its computational efficiency. The
refraction correction is done via maps, i.e., simple look-up tables for image
445 rectification which lead to very efficient operations very well suited for real-time
performance. The computations of the maps themselves is also relatively fast
and can be done just once offline. The following runtimes are benchmarked on a
Intel Core i7-3610QM CPU running at 2.3 GHz, i.e., a mobile CPU that is used

in an embedded system suited for integration on robotic vehicles, respectively
 450 even within the camera system itself. The experiments are done once with
 MATLAB R2014a on Windows 7 and once with the Robot Operating System
 (ROS) Hydro on Ubuntu 12.04. Note that no optimization is used, especially
 no parallelization is employed. Both the computation of the correction map
 itself as well as its application for rectification can be easily speeded up by
 455 parallel computation, e.g., through multithreaded or CUDA programming if
 higher processing speeds are required.

Table 2: Computation times for generating the correction maps.

camera	resolution	time	
		MATLAB (h:mm:ss)	ROS (mm:ss)
Bumblebee2	1024x768	0:11:47	0:20
Bumblebee XB3 / Vortex cam	1280x960	0:18:25	0:32
GoPro Hero3+ Black Ed.	4096x2160	2:12:36	3:50
Fuji FinePix 3DW3	3648x2736	2:29:35	4:20

Tab. 2 shows the computation times of the correction maps for different cam-
 eras, respectively image resolutions. The computation is linear in the number of
 pixels and takes about 0.89925 msec/pixel on MATLAB, respectively 0.026042
 460 msec/pixel on ROS. The computation of a pinax correction map has only to be
 done once. It can hence be simply computed offline.

Each pinax correction map depends - in addition to the in-air calibration map
 of the underlying physical camera - on the water refraction index, i.e., especially
 salinity. In the experiments reported later on, only two different correction maps
 465 are used across a wide range of different field experiments, namely one for salty
 water and one for sweet water. As discussed in more detail in the according
 sections, we found two maps to be sufficient. But for even more accurate image
 rectifications, it is possible to use a CTD sensor to determine the salinity of
 the water directly at the location of the mission, respectively even during the

470 mission if the salinity changes. This then allows to either instantaneously switch between several pre-computed maps, respectively to even compute a perfectly fitting map online, which takes in the order of at most a few minutes under ROS. The underlying algorithm is also well suited for parallel computation allowing faster processing if needed through multithreading or CUDA programming.

Table 3: Computation times for applying the correction maps, i.e., for rectification.

camera	resolution	time	
		MATLAB (seconds)	ROS (seconds)
Bumblebee2	1024x768	0.025	0.007
Bumblebee XB3 / Vortex cam	1280x960	0.055	0.012
GoPro Hero3+ Black Ed.	4096x2160	0.412	0.085
Fuji FinePix 3DW3	3648x2736	0.453	0.094

475 Tab. 3 shows the computation times to apply the correction maps, i.e., to perform an image rectification, for different cameras, respectively image resolutions. The computation is just a look-up operation and hence very fast and very well suited for real-time operation. The underlying algorithm is again also well suited for parallel computation; hence it is easy to further speed it up if
480 necessary through multithreading or CUDA programming.

4.3. Qualitative Results

In this section, we report qualitative results with a range of different under-water camera systems that illustrate the usefulness of the method introduced in this article for real world applications. Tab. 4 gives an overview of seven
485 different systems where the pinax-model was used on, i.e., the cameras in each system were calibrated just once in air and the pinax correction tables were used for rectification of the images. The correction tables were computed with two different refraction indices, namely $n_w = 1.333$ for sweet water, respectively $n_w = 1.342$ for salty water. Depending on the environment conditions,
490 e.g., experiments in a pool or lake, respectively in the sea, the according map

Table 4: The different camera systems used in qualitative evaluations

	camera	focal length (mm)	imager		housing provider
			resolution	size	
1	Bumblebee XB3 (Jacobs)	3.8	1280x960	1/3"	U.Zagreb
2	Bumblebee XB3 (IST)	3.8	1280x960	1/3"	U.Zagreb
3	Bumblebee2 (UdG)	2.5	1024x768	1/3"	UdG
4	Bumblebee2 (Jacobs)	2.5	1024x768	1/3"	ATLAS
5	Vortex Camera	4	1280x960	1/3"	Ifremer
6	Fuji FinePix 3DW3	6.3	3648x2736	1/2.3"	FantaSea
7	GoPro Hero3+ Black Ed.	2.65	4096x2160	1/2.3"	GoPro

was chosen. The cameras have different technical parameters, especially with respect to focal length or d_1 , and they are mounted in different housings that were all designed by third parties without any knowledge of the pinax model.

The test systems are all stereo cameras. The advantage of stereo cameras
495 is in this context that they not only provide metric information, which will be used for a quantitative analysis later on, but that their data also provides very good qualitative indicators of the calibration and rectification accuracy. Stereo processing is very sensitive to the accuracy of the image rectification due to the inherent use of the epipolar constraint. If there are distortions in the two
500 cameras, matching pixel blocks do not lie on the same line in the two images anymore, i.e., the epipolar constraint is violated, and correspondences can not be established leading to missing range values. Hence, rectification errors not only lead to metric errors in the range estimates but also to complete failures in the stereo computations.

505 The trials with different camera systems in different environment conditions show following main three qualitative results that are interesting for applying the method introduced in this article in real world applications:

1. *In-air calibration of underwater cameras with the pinax model is applicable to a range of systems and environment conditions.* We applied the method

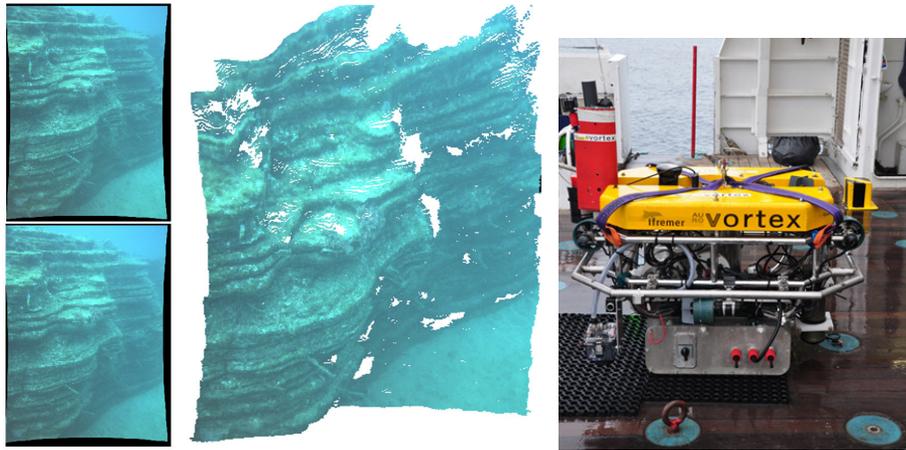


Figure 14: A 2.5D colored point cloud (center) generated from images (left) from a custom-made underwater stereo camera on the Vortex vehicle (right) of the Institut français de recherche pour l'exploitation de la mer (Ifremer). The stereo point cloud is very dense, hence indicating a very good rectification accuracy.

510 to seven different systems used in different environment conditions. The cameras and housings were from various 3rd parties. In each case, in-air calibration with the pinax model was successful and lead to (at least) qualitatively comparable results to underwater calibration which was the previous state of practice for the systems.

515 2. *The quality of the in-air calibration matters.* The pinax-model allows for convenient in-air calibration that only has to be done once. The final result of the rectification is significantly influenced by the quality of this calibration.

520 3. *The water refraction index, especially due to salinity, matters but to a lesser extent.* Ignoring the influence of the changes in refraction of water due to environmental parameters, especially in form of salinity, leads to a degradation in accuracy in the rectification.

Regarding aspect 1., the pinax model was successfully used on all seven systems. The in-air calibration and the related image rectification lead to high
 525 quality results in all cases as indicated by the density of the 3D point clouds



Figure 15: A 3D map generated from 350 stereo based point clouds from a Bumblebee camera on a Sparus vehicle of the University of Girona. The consistently dense stereo data in all stereo computations indicates high rectification accuracy.



Figure 16: A Point Grey Bumblebee 2 stereo camera and Blueview MB2250 Multibeam Echosounder integrated on a Seacat from ATLAS Elektronik.

generated by the stereo processing. Fig. 14 and 15 show two typical results as illustrative examples. Note that the “holes” in the point cloud shown in the center of Fig. 14 are just due to the perspective view, i.e., due to occlusions in the scene. In addition to the density of the stereo results, there are also
 530 qualitative indications of the metric accuracy. For example, Fig. 17 shows data from a Seacat vehicle from ATLAS Elektronik where a Point Grey Bumblebee 2 stereo camera and Blueview MB2250 Multibeam Echosounder (MBES) are integrated (Fig. 16). The range estimates of the two sensor systems are very close to each other, i.e., that the metric accuracy of the pinax-calibrated cameras
 535 is in the order of the MB2250 MBES.

Fig. 18 illustrates the aspects 2. and 3. with respect to the relevance of the quality of the in-air calibration, respectively of the water refraction index. The point cloud PC_c shown on the right was generated in seawater by system 2 (Bumblebee XB3 (IST) with U.Zagreb housing) using our method with the
 540 proper factory in-air calibration file as input and our standard salt-water refraction estimation. The resulting point cloud density ∇PC_c provides a comparison baseline for a simple illustrative example.

The point cloud PC_a shown on the left uses the factory in-air calibration

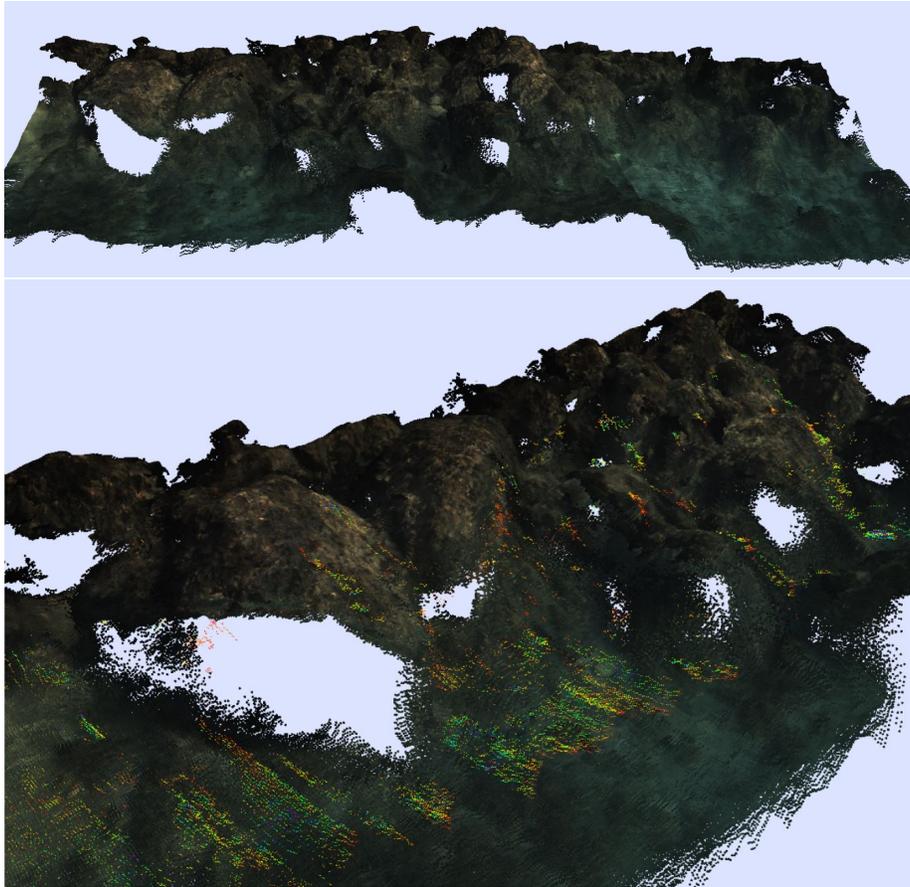
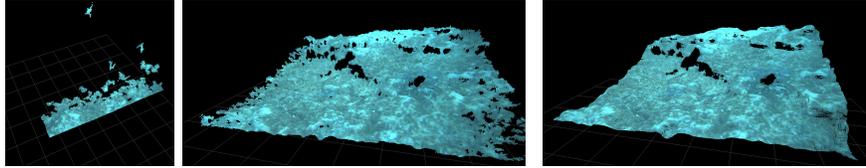


Figure 17: A 3D map with the multibeam data (shown in rainbow colors to indicate depth values) and the stereo data from the ATLAS Seacat vehicle (Fig. 16). The range estimates from the two different sensors are very similar, thus indicating that the metric accuracy of the pinax-calibrated cameras is in the order of the multibeam echosounder.



(a) wrong in-air (b) wrong salinity (c) correct in-air and salinity

Figure 18: An illustration of two of the three main qualitative observations related to our method, namely that 2. the quality of the in-air calibration matters (a) compared to c)) and that 3. the salinity has an influence (b) compared to c)).

file from exactly the same type of camera, namely the Bumblebee XB3 owned
 545 by Jacobs with identical (nominal) parameters as the one owned by IST, and
 which is mounted in the same type of housing, namely the design by U.Zagreb.
 The proper salt-water refraction index is used. Nevertheless, the point cloud
 density ∇PC_a is just 19.7% of the density ∇PC_c . So, no correspondences can
 be found for a significant portion of the pixels in both images, i.e., the necessary
 550 epipolar constraint for stereo vision does not hold, respectively the rectification
 process is highly unsuccessful in this case.

The point cloud PC_b shown in the center uses the correct factory in-air
 calibration file of this specific camera instance. But our standard sweet-water
 refraction index is here used in the pinax model though the data is collected
 555 in seawater. The point cloud density ∇PC_b degrades therefore to 93.2% of
 the density ∇PC_c in this example. It can be noticed that there is especially
 missing data at the sides of the point cloud, which is consistent with what is
 to be expected when the rectification quality degrades. The distortion effects
 due to refraction are most pronounced at the sides of the stereo images, hence
 560 violations of the epipolar constraint due to degraded rectification start taking
 effect from there.

4.4. Quantitative Evaluation of the Pinax Accuracy

The numerical analysis of the pinax model as well as the qualitative ex-
 periences in the field indicate that it leads to very accurate calibration and

565 rectification results. This is now further substantiated with quantitative evaluations of real cameras, namely a Bumblebee XB3 (Tab. 4, system 1) and a stereo rig consisting of two GoPro Hero3+ Black Edition (Tab. 4, system 7). Both systems are quite different and provide two interesting test cases.

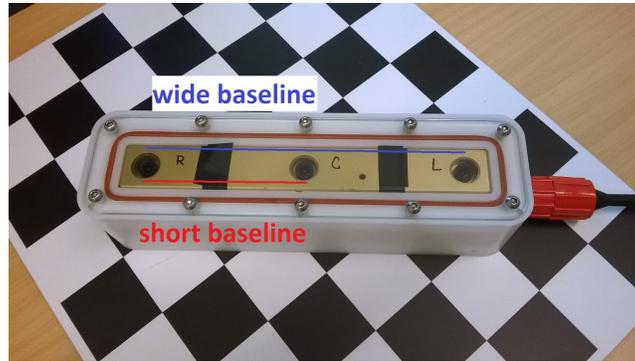


Figure 19: The Point Grey Bumblebee XB3 has three monocular cameras that allow stereo processing with a short and with a wide baseline. The checkerboard pattern underneath the camera is used for the quantitative accuracy analysis.



Figure 20: The stereo system consisting of two Gopro Hero3+ Black Edition cameras in a Gopro Dual HERO underwater housing.

The Bumblebee XB3 features three monocular cameras. This allows stereo
570 processing with a short and with a wide baseline (Fig. 19). The GoPro stereo system consist of a standard set-up with two cameras (Fig. 20). There are hence five monocular cameras in total that are calibrated and rectified with the pinax

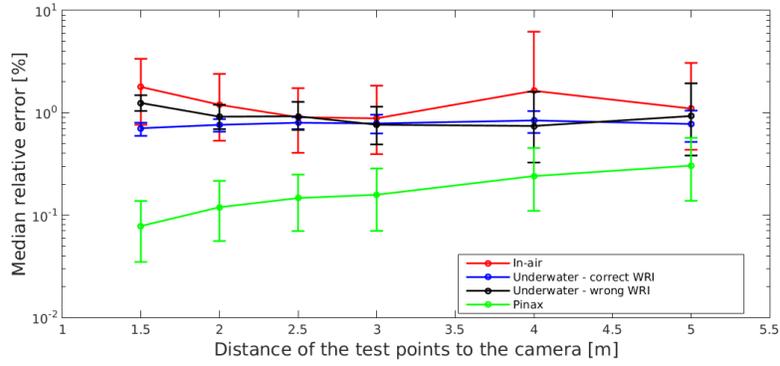
model in the following experiments.

As it is difficult or even impossible to acquire ground truth data of natural underwater environments, the analysis is based on artificial checkerboard patterns where the exact distance between the black and the white fields is accurately known. For the quantitative evaluations, the stereo systems are placed in a pool in which sweet, respectively salt water is filled. The checkerboard pattern is then moved at different distances within the field of view of each camera. Stereo processing is conducted for each sequence of images acquired at the different distances. The metric stereo estimates of the distances between the checkerboard markers are finally compared to the ground truth distances, thus providing an error metric for the rectification accuracy.

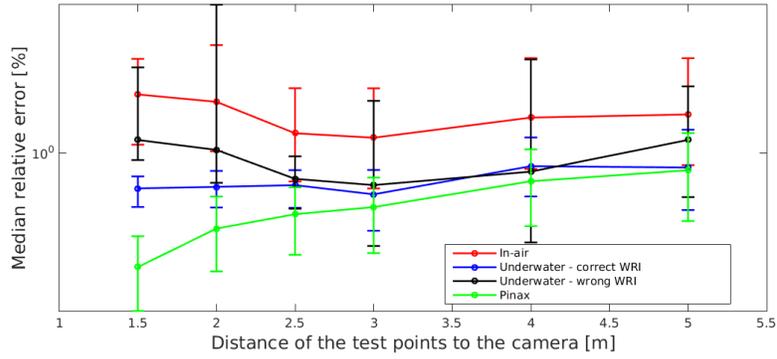
Four different methods for calibration and rectification are evaluated, namely:

- standard *in-air* calibration and rectification with a pin-hole model
- state of the art *underwater* calibration with a *correct* water refraction index (WRI), i.e., the calibration is performed in-situ in water at exactly the same salinity conditions as the recording of the evaluation data that is then rectified
- state of the art *underwater* calibration with a *wrong* WRI, i.e., the calibration is performed in a sweet water pool while the recording of the rectified evaluation data is done in salty water
- *pinax* in-air calibration and rectification under arbitrary but roughly known (sweet or salty) water conditions

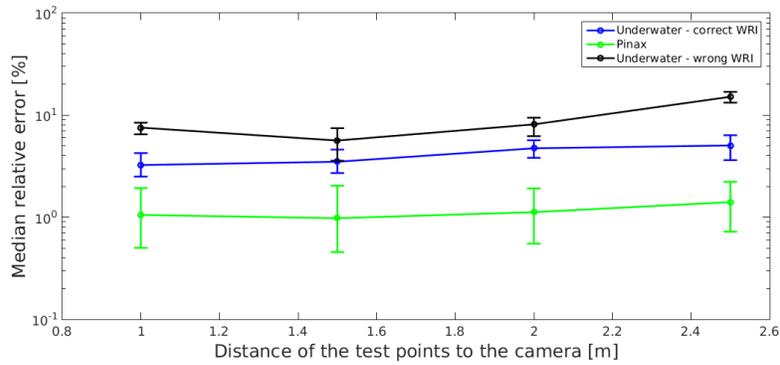
Fig. 21 shows the results of the evaluations on the three different stereo set-ups. All errors are normalized, i.e., they are plotted as percent of the known, measured distance between checkerboard markers. For the GoPro test sequence, the evaluation of the in-air calibration is omitted as its rectification is performing so poorly that stereo processing is not possible anymore (Fig. 22). Also in the case of the two Bumblebee XB3 set-ups, stereo processing for several of the recorded image pairs could not be performed due to poor rectification



(a) Bumble XB3, wide baseline



(b) Bumble XB3, short baseline



(c) GoPro

Figure 21: The relative errors of the triangulated points in % for the four evaluated methods on the three different stereo set-ups. The error bars show the upper and lower quartile values of the error, the center dots are the medians. The error values are plotted for each nominal distance of the calibration pattern during the test.



Figure 22: For the GoPro data, the in-air calibration leads to such severe distortions that the stereo processing is completely failing in this case. On the left, an example GoPro image from a test sequence is shown; on the right, the “corrected” image based on in-air calibration is shown.

results with the in-air calibration. These cases would have accordingly lead to significant metric errors; the reported average errors for the in-air calibration are hence a very optimistic, best case estimates.

605 It can be seen that this quantitative evaluation supports the previous numerical and qualitative observations. The pinax calibration and rectification leads in all cases to superior results. Most importantly, the errors are significantly smaller than using the state of the art underwater calibration. In addition, pinax calibration is much more conveniently to use as it is based on in-air calibration.
 610 tion. The experiments also show that the salinity matters, i.e., if state of the art underwater calibration is for example done in a sweet water pool and the camera is used in the sea, the rectification quality degrades. The pinax model takes the possible changes of the water salinity into account and is hence not affected by this.

615 5. Conclusions

In this article, the pinax model for calibration and rectification of underwater cameras in flat-pane housings was introduced. Its water/window refraction correction was derived from an analysis of the axial camera model for underwa-

ter cameras, which is among others computationally hard to tackle. Therefore,
620 realistic constraints on the distance of the camera to the window are used in the
pinax model to combine aspects of a virtual pinhole model with the projection
function from the axial camera model. It was shown how the pinax model al-
lows the pre-computation of a lookup-table for very fast refraction correction of
the flat-pane with high accuracy. The pinax model is very convenient to use in
625 practice as it is based on a single, in-air calibration of the complete underwater
camera system. The model takes the water refraction index into account for
which a rough estimate of the salinity (sweet/salty) is sufficient, respectively for
which in-situ measurements from e.g. a CTD-sensor can also be used to derive
perfectly fitting correction maps. The pinax model was analysed in this arti-
630 cle in three different ways. First, numerical simulations were used to illustrate
the influences of the different parameters on the accuracy and to motivate the
model. Second, qualitative results from use-cases of the model in the field were
presented that illustrated that it worked fine with various real world systems
designed by various third parties. Third, a quantitative analysis on different
635 stereo systems was performed where the pinax model lead a higher accuracy
than state of the art underwater calibration.

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